15

Analysis of Spinning Ball Trajectories of Soccer Kicks and Basketball Throws

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15.1 Scope

It is a common observation that putting spin to a pitched baseball or to a kicked soccer-ball imparts an out-of-plane curve to the ball trajectory. This mechanism has been successfully used in soccer to deceive goalkeepers, in basketball throws to obtain a better entering angle into the net, and in baseball to induce the batter to swing and miss. So then, what is the mechanism of this curving soccer kick or a backspinning basketball throw? In addressing this issue, this chapter deals with the mechanics of a spinning ball trajectory, as to how deviations in the ball’s original trajectory are governed by different amounts of angular velocity and translational velocity. It is shown as to how the amount and direction of spin plays an important role in delineating the trajectory and the position of the ball.
in its trajectory. Then, this analysis is employed to simulate a number of interesting situations in soccer and basketball, including the famous Ronaldinho’s goal against England in the 2002 World-cup semifinals.

### 15.2 Theory: Lateral Force on the Spinning Ball of a Soccer Kick

When a soccer ball is kicked with a spin about the vertical axis of the ball, it will swerve laterally from its vertical–planar trajectory because of the lateral force exerted by the air on the ball, as illustrated in Figure 15.1a through c.

In Figure 15.1, the ball is kicked from right to left at velocity $u$. The air is flowing in the opposite direction from left to right; $u_\infty$ is the horizontal velocity of air, $\omega$ is the counter-clockwise angular velocity imparted to the ball, $\Gamma$ is the Circulation, and $u$ is the horizontal velocity of kick. In Figure 15.1, it is noted that the ball has a higher velocity on one side (Figure 15.1b), and a higher pressure intensity on the opposite side (Figure 15.1c), which results in a transverse force as shown in Figure 15.1c. This lateral or transverse force makes the ball curve sideways, as it travels from right to left in the air.

Also, if the ball is kicked and spun in a vertical plane, it will have a lift force (upward or downward) resulting from a combination of angular and translational velocities caused by the air. As seen in Figure 15.1, the airflow on one side of the ball is retarded relative to that on the other side, and the pressure becomes greater than that on the other side, so the ball is pushed laterally. The greater the velocity of the spin, the larger is this lateral

![Figure 15.1](image)

**FIGURE 15.1**

Effect of circulatory flow superimposed on translatory flow. Here, the ball is kicked to the left (as shown in (a)) and the spin imparted to it is counter-clockwise (as shown in (b)). The resultant airflow pattern (c) causes a transverse force on the ball, causing the ball to curve sideways as it travels from right to left.
force or lift force. This phenomenon of lateral or lift motion produced by imposing circulation over a uniform fluid stream, known as the Magnus effect, can be used to explain the deviation of spinning balls from their normal trajectories.

The corresponding lift force or lateral force (Figure 15.1) can be calculated by Equation 15.1 derived from the Kutta–Joukowski Law, being first noted by the German physicist, H.G. Magnus (1802–1870) and in honor of the German and Russian fluid dynamists M.W. Kutta (1867–1944) and N.E. Joukowski (1847–1927). They independently showed that, for a body of any shape, the transverse force per unit length is \( \rho u_1 \Gamma \), and is perpendicular to the direction of the air velocity \( u_1 \). The formula for this lateral or transverse force (acting on the spinning ball) is given by

\[
F_L = \frac{1}{2} \rho \pi R^3 \omega v_{0h} \quad (15.1)
\]

where
- \( F_L \) is the lateral force
- \( \rho \) is the air density
- \( R \) is the radius of the ball
- \( \omega \) is the angular velocity of the spin
- \( v_{0h} \) is the initial horizontal velocity

A brief derivation of this expression is provided in the Appendix, while some interesting applications of this phenomenon are given in the references [1–5].

Using this simplified equation for lateral or lift force, we have studied different cases of balls spinning with different velocities. Our results show that the angular velocity of the ball during its motion in the air causes the ball to deviate from its original trajectory, by different amounts for different angular velocities and initial translatory velocities. Hence, the amount and direction of spin play an important role in deciding the trajectory and final position of the ball.

In Figure 15.2, a case study of such a soccer kick is illustrated, to be analyzed later in Section 15.3.3. It is seen that the spin of the ball contributes about 6.10 m in the direction of \( y \), which results from the acceleration in \( y \) direction caused by the lateral (or transverse) force due to spin or angular velocity (\( \omega \)) imparted to the ball. In the feasible ranges of the angular velocity or spin imparted to the ball and the initial velocity with which the ball is kicked, the deviation (because of the spin) can vary from 2 to 5 m in the normal kicking range. It is no wonder that a corner kick, if properly taken with the right combination of initial velocity \( v_0 \) and \( \omega \), can make the ball swerve into the goal just under the bar and place the ball in the top-far corner of the goal (as illustrated later on in Figure 15.5).
Note, herein, that Ronaldinho’s “wonder kick” made the ball appear to go over the bar and curve back under the bar into the far-top corner of the goalpost. No goalkeeper could have stopped it.

FIGURE 15.2 (See color insert following page 266.)
Analytical simulation of the trajectory of Ronaldinho’s famous free kick in the quarter-final match against England in the 2002 World Cup (won by Brazil). The top figure shows the 3-D trajectories of the ball, with and without spin. The bottom figure shows the top view (or the horizontal projection) of the ball trajectory to its final location B into the goal. In doing so, to the goalkeeper Seaman, the ball must have actually appeared to be sailing over the bar, only to see it curve back to dip below the bar into the goal. In the figure, BC represents the goal bar, \( k \) is the unit vector making an angle \( \theta \) with the \( x \)-axis, and \( \beta \) is the angle that the initial velocity vector \( \mathbf{v}_0 \) makes with \( \mathbf{k} \) (in the \( xOz \) plane). The initial velocity vector \( \mathbf{v}_0 \) lies in the \( zOx \) plane. The lateral deviation of the ball along the goal bar is 6.10 m, as shown in the figure.
15.3  Analysis of the Soccer Kick

15.3.1  Theory: Trajectory of a Spinning Soccer-Ball Kick

As shown in Figure 15.3, the ball is kicked with an initial velocity \( v_0 \) at angle \( \beta \) with the horizontal in the \( zOk \) plane, making an angle \( \theta \) with the \( zOx \) plane. The conventional governing equations, of the ball’s \( (x, y, z) \) displacement-time relations without spin, are

\[
x = (v_0 \cos \beta) \cos \theta t \\
y = (v_0 \cos \beta) \sin \theta t \\
z = v_0 \sin \beta t - \frac{1}{2} gt^2
\]

where

\( v_0 \cos \beta \) \( (= v_{ok}) \) is the horizontal component of the initial velocity along \( Ok \)

\( OE = (v_{ok})t \) is the distance covered along the horizontal axis \( Ok \) (on the ground)

\( g \) is the gravity acceleration

Now, when the ball is kicked by imparting it spin, i.e., angular velocity \( \omega \) (counter-clockwise, looking down on the ball) about the vertical axis through the ball (refer to Figure 15.4), then the ball displacement–time relations (in the \([k, j, z]\) coordinate frame) are given by (refer to Figure 15.4)

\[
\text{OPD is the ball trajectory; this trajectory plane (OPD) slopes with respect to the vertical zOk plane.}
\]

\[
The ball is kicked in the zOk plane at an angle \( \beta \) to the horizontal plane or to the axis \( Ok \) in the horizontal plane. The axes \( Ok \) and \( Oj \) make angle \( \theta \) with the axes \( Ox \) and \( Oy \).
\]

\[
\text{Path of the ball viewed from above or as projected on the horizontal.}
\]

**FIGURE 15.3 (See color insert following page 266.)**

Notations for soccer-ball kick-velocity and trajectory. The orthogonal lines (or axes) \( Ok \) and \( Oj \) are in the horizontal plane \( xOy \), and make angles \( \theta \) with the \( Ox \) and \( Oy \) axes, respectively. \( ED \) is the total horizontal deviation \( (d) \) of the ball when it lands on the ground at \( D \). The curve \( ORD \) is the horizontal projection of the trajectory of \( OPD \).
Horizontal displacement along $Ok$ axis,
\[ k(=OE) = (v_0 \cos \beta)t = v_{ok}t \quad (15.5) \]

Ball displacement in the “$j$” direction (normal to $Ok$),
\[ j(=ED) = d \quad (15.6) \]

wherein $d$ is given by the following Equations 15.8 and 15.10

Vertical displacement along $Oz$,
\[ z = (v_0 \sin \beta)t - \frac{1}{2}gt^2 \quad (15.7) \]

wherein
\[ d = \frac{1}{2}a_L t^2 \quad (15.8) \]

with $a_L$ (given by Equation 15.10) being the lateral acceleration due to the lateral force (caused by $\omega$).

From Equation 15.5
\[ t = k(\text{displacement})/v_{ok} = \frac{OE}{(v_0 \cos \beta)} \quad (15.9) \]

with $\beta$ being the angle of the tangent to the trajectory with respect to the horizontal $Ok$ axis.

---

**FIGURE 15.4** (See color insert following page 266.)
Ball displacements in the horizontal plane. The ball is kicked in the $zOk$ vertical plane. However, because of the counter-clockwise angular velocity ($\omega$) imparted to it, it has deviated by an amount “$d$” ($=ED$) perpendicular to $Ok$ (i.e., parallel to $Oj$ axis) when it lands on the ground.
The lateral acceleration

\[ a_L = \frac{F_L}{m} \]

\[ = \frac{1}{2m} \rho \pi R^3 \omega v_{ok} = \frac{1}{2m} \rho \pi R^3 \omega (v_0 \cos \beta) \]  \hspace{1cm} (15.10)

where

- \( F_L \) is the lateral force (Equation 15.1)
- \( m \) is the mass of the ball

Now, in the \((x, y, z)\) coordinate frame, the ball displacement–time relations are given by (Figures 15.3 and 15.4)

Ball displacement along \(Ox\) axis,

\[ x = [(v_0 \cos \beta) \cos \theta]t - d \sin \theta \hspace{1cm} (15.11) \]

Ball displacement in the \(Oy\) direction,

\[ y = [(v_0 \cos \beta) \sin \theta]t + d \cos \theta \hspace{1cm} (15.12) \]

Vertical displacement of the ball,

\[ z = (v_0 \sin \beta)t - \frac{1}{2}gt^2 \hspace{1cm} (15.13) \]

where \( d \) is given by Equation 15.8, and based on Equation 15.9

\[ t = \frac{k \text{(displacement) or } OE}{v_0 \cos \beta (= v_{ok})} = \frac{(x + d \sin \theta) / \cos \theta}{v_{ok} (= v_{ox} / \cos \theta)} = \frac{x + d \sin \theta}{v_{ox}} \]

\[ = \frac{OF}{v_{ox}} = \frac{x'}{v_{ox}} \hspace{1cm} (15.14) \]

The resulting trajectory (OPD) of the ball is shown in Figure 15.3, along with the curved horizontal projection of the ball trajectory (ORD) in the horizontal plane. D is the point at which the ball lands on the ground and P is the highest point of the ball trajectory.

Let us take some reasonable data: ball radius \( R = 10 \) cm, \( \omega = 30 \) rad/s, distance traveled along \( OOk(=v_{ok}) = 40 \) m, initial velocity \( v_0 = 23 \) m/s, angle \( \beta \) of the initial velocity vector with respect to the axis \( OOk = 20^\circ \), air density \( \rho = 1.25 \) kg/m\(^3\), ball mass \( m = 0.5 \) kg. If we substitute these data into Equations 15.5 through 15.14, then
From Equation 15.14, the time taken for the ball to land, \( t = \frac{40}{23} \cos \beta = 1.85 \) s.

From Equations 15.8 and 15.10, the ball deviation \( (d = DE) \) perpendicular to \( Ok \) in the \( kOj \) plane, when it lands (= ‘‘j’’ displacement of the ball)

\[
\frac{\rho \pi R^3 \omega (v_0 \cos \beta) t^2}{4m} = 4.35 \text{ m}
\]

Distance OD traveled by the ball in the horizontal plane (Equations 15.5, 15.6, and 15.15)

\[
(k^2 + j^2)^{1/2} = (OE^2 + ED^2)^{1/2} = 40.24 \text{ m}
\]

The \( z \) coordinate of the highest point \( P \) of the trajectory is obtained by putting the \( z \) (or vertical) velocity of the ball \( (v_0 \sin \beta - gt) = 0 \), and substituting \( t \) (time taken for the ball to go from \( O \) to \( P \)) = \( v_0 \sin \beta / g = 0.8 \) s into Equation 15.7, to obtain \( z(P) = 3.16 \) m.

The \( (k, j) \) coordinates (in the coordinate plane \( kOj \)) of the point \( R \) (the projection of the ball-trajectory point \( P \) on the horizontal plane) = (17.29, 0.81)

15.3.2 Exemplification of the Theory: Computation of the Spinning Ball Trajectories

We will now solve some realistic soccer situations.

Example 15.1

Let us analyze how a spinning kick from the goal line can make the ball swerve into net, as illustrated in Figure 15.5.

Here, we are given the final coordinates of the ball to be at \( C (41, 0, 2.4) \), the top-far corner of the goal. The ball is kicked by a right footer in the vertical plane \( zOk \) at \( v_0 = 28 \) m/s, \( \beta \) (angle made by \( V_0 \) with \( Ok \)) = 19°, ball mass \( m = 0.5 \) kg, ball radius \( R = 0.1 \) m, air density \( \rho = 1.25 \) kg/m³. We need to determine the values of \( \theta \) (illustrated in Figure 15.5), \( t \) and \( \omega \), such that the ball will land in the top-far corner of the goalpost.

The governing relations are (as adopted from Equations 15.5 through 15.14): displacement along \( Ok \), distance \( k \) (Equation 15.5)

\[
= (v_0 \cos \beta) t = v_{Ok} t = OE = OD \cos \theta
\]

deviation, \( d \) (Equations 15.8 and 15.10)

\[
= \frac{\rho \pi R^3 \omega v_0 t^2}{4m} = \frac{\rho \pi R^3 \omega (v_0 \cos \beta) t^2}{4m} = ED = OD \sin \theta
\]
final vertical displacement $z$ (Equation 15.13) = $(v_0 \sin \beta)t - \frac{1}{2}gt^2 = DC$

$$t = \frac{k(\text{distance})}{v_0 k} = \frac{OE}{v_0 k} \text{ (horizontal velocity in the } k \text{ direction)}$$

These relations are rewritten as

$$OE = (v_0 \cos \beta)t = OD \cos \theta = x(D) \cos \theta, \text{ where } OD = 41 \text{ m (as per data)}$$

$$ED(=d) = \left(\frac{\rho \pi R^3 \omega v_0 \cos \beta}{4m}\right)t^2 = OD \sin \theta$$

$$DC = (v_0 \sin \beta)t - \frac{1}{2}gt^2 = 2.4 \text{ m}$$

These 3 equations have to be solved for $t$, $\theta$, and $\omega$. From Equation 15.23, we get $t = 1.54 \text{ s}$. Then, from Equation 15.21 and 15.22, we get

$$(v_0 \cos \beta)^2 t^2 + \left(\frac{1}{4m} \rho \pi R^3 v_0 \cos \beta\right)^2 \omega^2 t^4 = OD^2$$

Putting $t = 1.54 \text{ s}$, $v_0 = 28 \text{ m/s}$, $\beta = 19^\circ$, $m = 0.5 \text{ kg}$, $R = 0.1 \text{ m}$, $\rho = 1.25 \text{ kg/m}^3$, and $OD = 41 \text{ m}$ into Equation 15.24, we obtain $\omega = 35.18 \text{ rad/s}$. Finally from Equation 15.21, we compute $\theta = 6.06^\circ$. So we have shown that if the ball is kicked with an initial velocity of 28 m/s and angular velocity of $35 \text{ rad/s}$ at $\theta = 6^\circ$ and $\beta = 19^\circ$, the ball will swerve into the goal.
On October 16, in the 11th min of the Euro 2004 qualifier match between England and Macedonia, Macedonia’s Artim Sakiri carried out this feat, as illustrated in Figure 15.6.

Example 15.2
To plan a freekick from top of the box, by making the ball bend around the players’ wall (Figure 15.7).

Case 1: Right-footer kick, in the \( zOk \) vertical plane with \( \omega = \omega_1z \) (Figure 15.7).

We adopt coordinates of the final location of the ball top-near corner of the goal-net point \( C = (16.5, 5.5, 2.4) \), initial velocity \( v_0 = 14 \text{ m/s} \), angle of the

FIGURE 15.6
Illustration of Sakiri’s corner kick into the far-top corner of the goalpost, as simulated in Example 15.1.

FIGURE 15.7 (See color insert following page 266.)
Left-footer kick (OB) and right-footer kick (OC) around the players’ wall into the goalpost. The ball is kicked in the vertical planes \( zOs \) and \( zOk \).
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initial velocity vector with the horizontal $\beta = 46^\circ$, air density $\rho = 1.25 \text{ kg/m}^3$, ball radius $R = 0.1 \text{ m}$, ball mass $m = 0.5 \text{ kg}$.

We want to determine the angular velocity (or spin $\omega$) to be imparted to the ball and the angle $\theta$ of the vertical plane $zOk$ with the plane $zOx$, such that the ball ends up at $C$ into the top corner of the goal. For this purpose, we first solve Equations 15.11 through 15.13, in which there are 3 unknowns, namely $d$, $\theta$, and $t$.

In Equation 15.13, by substituting $z = 2.4 \text{ m}$ and the data values of $v_0 = 14 \text{ m/s}$ and $\beta = 46^\circ$, we get $t = 1.78 \text{ s}$.

From Equations 15.11 and 15.12,

$$x(D) \cos \theta + y(D) \sin \theta = OE \quad \text{(in Figures 15.7 and 15.4)} = (v_0 \cos \beta)t \quad \text{(15.25)}$$

By substituting $x(D) = 16.5 \text{ m}$, $y(D) = 5.5 \text{ m}$, $t = 1.78 \text{ s}$, $v_0 = 14 \text{ m/s}$, and $\beta = 46^\circ$, we get $\theta = 12.92^\circ$.

Also, from Equations 15.11 and 15.12, we can put down

$$y \cos \theta - x \sin \theta = d,$$

from which we get $d = 1.67 \text{ m}$.

Then, from Equations 15.8 and 15.10, we get (for $d = 1.67 \text{ m}$ and $t = 1.78 \text{ s}$) $\omega = 27.69 \text{ rad/s}$.

Case 2: Left-footer kick, in the $zOs$ vertical plane, with $\omega = -\omega i_z$ (Figures 15.7 and 15.8).

We need the coordinates of (the far-top corner of the goal-net) point B ($= 16.5, 12.8, 2.4$) in Figure 15.7 and point A ($= 16.5, 12.8, 0$) in Figure 15.8. We take $v_0 = 16 \text{ m/s}$, $\beta = 61^\circ$, $\rho = 1.25 \text{ kg/m}^3$, $R = 0.1 \text{ m}$.

**FIGURE 15.8** (See color insert following page 266.)

Geometry of the left-footer kick in the horizontal $zOxy$ plane: $zOs$ is the vertical plane in which the ball is kicked, $TA (d)$ is the horizontal deviation of the ball trajectory, $A$ is the horizontal projection of the ball-location B (into the net).
The geometry of the kick and ball trajectory is illustrated in Figure 15.8. The \((x, y)\) coordinates of the ball are now given by the following equations (instead of Equations 15.11 and 15.12):

\[
x = [(v_0 \cos \beta) \cos \theta] t + d \sin \theta \\
y = [(v_0 \cos \beta) \sin \theta] t - d \cos \theta
\] (15.27) (15.28)

For the vertical displacement of the ball, we employ Equation 15.7

\[
z = (v_0 \sin \beta) t - \frac{1}{2} gt^2
\] (15.29)

The deviation of the ball trajectory is given by Equations 15.8 and 15.10, as

\[
d = \frac{1}{2} a t^2 = \frac{\rho \pi R^3 \omega (v_0 \cos \beta) t^2}{4m}
\] (15.30)

From Equation 15.29, by substituting \(z = 2.4\) m and \(\beta = 61°\), we get \(t = 2.67\) s. From Equations 15.27 and 15.28,

\[
x(A) \cos \theta + y(A) \sin \theta = OT = (v_0 \cos \beta) t = 20.73
\] (15.31)

By substituting \(x(A) = 16.5\) m and \(y(A) = 12.8\) m, we get \(\theta = 44.70°\).

Similarly, from Equations 15.27 and 15.28,

\[
x(A) \sin \theta - y(A) \cos \theta = d(TA) = \frac{\rho \pi R^3 \omega (v_0 \cos \beta) t^2}{4m}
\] (15.32)

By substituting, in Equation 15.32, the value of \(x(A)\) and \(y(A)\), \(\rho, R, v_0, \beta, m\) (as provided earlier), we obtain \(\omega = 23.10\) rad/s, \(d = 2.51\) m. It would appear that the goalkeeper would find it more difficult to judge a spot kick taken by a left footer.

**15.3.3 Case Study: Analysis of the Famous Ronaldinho Goal against England in the Quarter-Final of the 2002 World Cup**

In the 2002 World Cup, Brazil won a free kick 30 m out on the right flank. Ronaldinho kicked the ball with his right foot and made the ball spin anticlockwise (view from top) (Figure 15.9). The shot was aimed at the far corner of the goal and the ball just dipped under the bar. This wonder goal won the game for Brazil. So let us simulate his goal, by doing an inverse analysis. Obtaining the initial shot angles \(\beta\) and \(\theta\) by reviewing the video tape, we carried out an inverse analysis, by computing the values of the angular velocity (\(\omega\)), the initial velocity \((v_0)\), and the in-flight time \(t\) of the ball (Figure 15.2).

From the video, we estimated, \(\theta = 38.45°, \beta = 43°\), and the coordinates of the ball landing into the goal (at the top right corner B) to be: \(x(B) = 30,\)
Brazil won a free kick when Scholes tackled Kleberson from behind, 30 m out on the right flank. Five Brazilians lined up across the edge of the penalty area, seemingly ready for the ball to be crossed towards the far-post.

Goalkeeper David Seaman obviously expected this too. He was only 3 m off his line and took a small step forward when Ronaldinho struck the free kick.

But Ronaldinho’s shot was aimed at the far-top corner and dipped just under the bar with Seaman flapping helplessly. In a few short minutes, Ronaldinho had turned the game on its head.

y(B) = 30, z(B) = 2.4. We now solve Equations 15.9 through 15.13, for the following unknown variables \( v_0, \omega = \omega_z \), and \( t \). On the basis of Figure 15.2, the relevant equations are

\[
\begin{align*}
    x(B) &= 30 = [(v_0 \cos \beta) \cos \theta]t - d \sin \theta \\
    y(B) &= 30 = [(v_0 \cos \beta) \sin \theta]t + d \cos \theta \\
    z(B) &= 2.4 = (v_0 \sin \beta)t - \frac{1}{2}gt^2
\end{align*}
\]

From Equations 15.33 and 15.34,

\[ x(B) \cos \theta + y(B) \sin \theta = (v_0 \cos \beta)t = 42.08 \]  

where \( \theta = 38.45^\circ \) and \( \beta = 43^\circ \).

From Equation 15.35,

\[ 2.4 = v_0 \times (\sin 43^\circ)t - \frac{1}{2}gt^2 \]  

**FIGURE 15.9 (See color insert following page 266.)**

Ronaldinho’s wonder goal, the famous right-foot free kick, that made the ball curve into the far-top corner of the goalpost and won the game for Brazil in the 2002 World-cup quarter-finals.
We then solve Equations 15.36 and 15.37, to obtain the values of $v_0 = 21 \text{ m/s}$ and $t = 2.74 \text{ s}$. Then based on Equations 15.8 and 15.10, the deviation from the vertical plane in which the ball was kicked (with angular velocity $\omega$), is computed from either Equation 15.33 or 15.34, as:

$$d = \left( \frac{\rho \pi R^3 v_0 \cos \beta t^2}{4m} \right) \omega = 4.84 \text{ m},$$

for geometrical construction of the trajectory (as illustrated in Figure 15.2), giving:

$$\omega = 21.35 \text{ rad/s}. \quad (15.38)$$

The deviation along the goal line is $d/cos \theta = 6.10 \text{ m}$. If the ball had not spun, it would have sailed over the top of the goal. Even if the ball had been kicked with a lesser initial velocity and a smaller angle $\beta$, it would not have curved that much, and it would have been easy for David Seaman (England’s goalkeeper) to grab the ball because at that time he stood just in the middle of the goal, as seen in Figure 15.9.

However, the tremendous angular velocity (of 21.35 rad/s) imparted to the ball made it swerve and sail over Seaman, making it impossible to reach it, despite of his height (as shown in Figure 15.10). From the analysis

FIGURE 15.10 (See color insert following page 266.)
Seaman tried to reach the ball but failed, and the ball just dipped below the bar into the top corner of the goal-net.
and computation, it can be seen that this wonder kick had the adroit combination of $\theta = 38.45^\circ$, $\beta = 43^\circ$, $v_0 = 21$ m/s, $\omega = 21.35$ rad/s, making the ball appear to come from outside and above and across the bar into the net.

### 15.4 Basketball Foul Throw Analysis

One can often see (in an NBA game) that when a player shoots a ball into the basket, the ball spins backwards when it leaves the hand of the player. Does this technique help to improve the accuracy (and stability) of the throw? The answer is yes. The backward spin produces an upward lift force, that enables the ball to have a higher trajectory and a bigger entering angle into the basket than a throw without spin. The advantage of a bigger entering angle (in Figure 15.11) is that it reduces the possibility of the ball hitting the rim and rebounding out, and makes the ball enter the basket squarely. Here, we have studied the case of a throw, taken from the foul line.

The following conventional equations of trajectory are used for describing the ball motion without spin (Figure 15.11):

\[
x = v_0 \cos \theta t \\
z = v_0 \sin \theta t - \frac{1}{2}gt^2
\]

(15.39) \hspace{1cm} (15.40)

where $\theta$ is the angle between the initial velocity and the horizontal direction (the attacking angle or the shooting angle).

However, if a backspin ($\omega$) is imparted to the ball, the corresponding equations for the spinning ball motion trajectory become altered to

\[
x = v_0 \cos \theta t \\
z = v_0 \sin \theta t + \frac{1}{2}(a_L - g)t^2
\]

(15.41) \hspace{1cm} (15.42)

where $a_L$, the lift acceleration (as a result of $\omega$) is given by Equation 15.10, as before in the case of a soccer ball.

Suppose in making a foul shot, as the ball leaves the player A’s hands, it is about 2.1 m above the ground, i.e., $z_0 = 2.1$ m, in Figure 15.12. Then as the ball travels in a vertical plane to enter the basket, it covers 0.95 m in the vertical direction ($z$ direction) and 3.97 m in the horizontal direction ($x$ direction). If a backspin is imparted to the ball when throwing, then because of $a_L$ (in Equation 15.42), the ball has a smaller acceleration towards the ground, due to the lift force generated by the spin.

In our simulation of player A’s throw, we found out computationally that shots with initial velocity of 7–8 m/s, would need to have angular velocity of backspin ranging from 1 to 10 rad/s (for different initial attacking angles to the horizontal plane) in order to be able to enter the basket. The two cases
studied (and illustrated in Figure 15.12) show that the ball with spin is lifted by the lift force compared to the ball without spin. The deviations in these two cases of $v_0 = 7$ and $8$ m/s are 0.09 and 0.05 m, respectively; the corresponding lift accelerations $a_L$ (computed from Equation 15.10) are $0.296$ m/s$^2$ and $0.35$ m/s$^2$, respectively. It also means that if a player wants the ball to enter the basket without spin, he has to throw the ball at a higher angle $\theta$ and a greater velocity $v_0$ to make the basket.

Our study also suggests that it is optimal for player A to throw the ball with an initial velocity of 7–8 m/s, with an angular velocity $\omega$ of 1–10 rad/s, and an initial shooting angle $(\theta)$ varying from $30^\circ$ to $65^\circ$. By throwing in this way, he can make the ball pass directly through the middle of the basket.
If he now wants to achieve the same result with a lower initial velocity, he would have to make the ball spin in the air at more than 100 rad/s, which is impossible in practice (if other parameters remain unchanged). On the other hand, if he would have to make a steeper throw with a higher velocity of throw, this is far more difficult to control. Hence for player A, the adroit combination of throw parameters to make a foul throw at an initial velocity \( v_0 \) of 7–8 m/s, is with a backspin angular velocity of \( \omega = 1–10 \text{ rad/s} \), at \( \theta = 30^\circ–65^\circ \).

### 15.5 Concluding Remarks

In a soccer kick or a basketball throw, there is bound to be some spin imparted to the ball because of the orientation of the hand and foot at the
time of releasing and making ball contact, respectively. This spin will alter
the trajectory of the ball in an unpredictable way. Hence, in order to shoot
accurately, it is better to impart a deliberate spin to the ball.

Herein, we have simulated some soccer situations involving corner kick
and free kick taken just outside the box, and demonstrated how a spin-
impacting kick can make the ball deceptively swerve into the goal. The type
of kick, making the swerve, can also be employed by a halfback to pass to a
forward or to a striker out of reach of the defenders. As regards corner kicks,
it needs to be recognized that a right footer taking the kick from the left
corner will make the ball swerve towards the goal, while a left footer taking
the kick from the left flank will make the ball swerve away from the goal.
The reverse holds good when the kick is taken from the right corner.

As regards basketball, it is often seen that tall forwards are not so prolific
as shorter guards in foul shooting. Part of the reason could be that their
greater height requires a flatter ball trajectory, which is more difficult.
However, if they were to impart backspin to the ball, it would make the
ball arch more, so as to easily enter the net.

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**Appendix**

Equation 15.1 is derived by Watts and Bahill [5]. They identified
three dimensionless parameters to describe the results of their lift-force
experiments:

\[
C_L = \frac{2F_L}{\rho A v^2}, \quad SP = \frac{R \omega}{v}, \quad Re = \frac{2\rho R}{v} \tag{15.A1}
\]

where

- \(C_L\) is the lift coefficient
- \(SP\) is the spin coefficient
- \(Re\) is the Reynolds number
- \(A\) is the cross-sectional area of the ball
- \(R\) is the radius of the ball
- \(v\) is the horizontal velocity
- \(\omega\) is the angular velocity of the spin
- \(\nu\) is the kinematic viscosity of the fluid
- \(\rho\) is air density

Watts and Bahill [5] plotted \(C_L\) versus \(SP\) for different types of balls, as
depicted in Figure 15.A1. Typical values of \(SP\) for baseballs range between
0.1 and 0.2. In this range, \(C_L\) varies almost linearly with \(SP\). Infact, for \(SP\) less
than 0.4, the relation between $C_L$ and SP is almost a straight line of slope unity. Hence, from $C_L / C_b$, we get

$$2F_L = \frac{R \omega}{\nu}$$

Substituting $A = \pi R^2$, and rearranging the equation yields:

$$F_L = \frac{1}{2} \rho \pi R^3 \omega \nu$$

Out of interest, it may be mentioned that the transverse force ($F_L$), on a cylindrical body (of length $L$ and radius $R$) is given by

$$F_L^C = \pi \rho_a (2LR) R \omega \nu = \pi \rho_a A^C R \omega \phi_{sb} = 2 \rho_a V^C \omega \phi_{sb}$$
where

\( v_\infty \) (the air-flow velocity away from the body) corresponds to the \( \theta_{\text{oh}} \)

the initial horizontal velocity imparted to the body

\( 2LR \) is the vertical area \((A')\) of the projection of the surface exposed to the flow

\( R \) is the radius of body cross-section

\( \pi R^2 L \) represent the body volume \((V^c)\)

On the other hand, for airflow past a spherical body, we have (from Equation 15.A3):

\[
F_L^S = \frac{1}{2\pi^{1/2}} \rho_a A_s^{3/2} \omega \theta_{\text{oh}} = \frac{3}{8} \rho_a V^S \omega \theta_{\text{oh}}
\]

(15.A5)

where

\( A_s \) is the projected area exposed to airflow

\( V^S \) is the volume of the spherical body

A comparison of expressions Equations 15.A4 and 15.A5 shows that \( F_L^C \) is about 5–10 times bigger than \( F_L^S \), as can be expected.

References